B.Sc. Semester-V Examination, 2022-23

PHYSICS [Honours]

Course ID : 52411 Course Code : SH/PHS/501/C-11

Course Title : Quantum Mechanics & Applications

Time : 1 Hour 15 Minutes Full Marks : 25

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

SECTION–I

1. Answer any **five** questions: 1×5=5

a) Consider a stationary state of energy E. Find how the wave function $\Psi_E(t)$ for this state varies with time.

b) What do you mean by group velocity and phase velocity?

c) Calculate $\left[x^n,\ p_x\right]$.

d) What is zero-point energy of a harmonic oscillator?

e) What is probability current density?

f) What are Stokes and Anti-Stokes lines?

g) Evaluate the commutation relation $\left[L_x,\ L_z\right]$.

h) Show that for a Hermitian operator, the eigenvectors corresponding to different eigenvalues are orthogonal.

SECTION–II

2. Answer any **two** questions: 5×2=10

a) Briefly describe the basic outcomes of the Stern-Gerlach experiment. Why the magnetic field used in this experiment is taken to be non-uniform? Why a beam of neutral atoms was used instead of using ions? Given that $\hat{A},\ \hat{B}$ are Hermitian operators, check whether $\left[\hat{A},\ \hat{B}\right]$ is Hermitian or not. 1+1+1+2

b) i) Show that if Ψ be an Eigen function of the operator $\hat{A}$ with Eigen value λ, then Ψ is also an Eigen function of e^A with Eigen value e^{λ}.

ii) A particle of mass m moves in a one-dimensional square potential well with infinite potential barriers located at $x = 0$ and at $x = L$. Find out the allowed energy

values and the eigen functions corresponding to the eigen values. How does the energy level change if the length of the well is doubled? 2+(2+1)

c) Write down Schrödinger's equation for the electron of Hydrogen atom assuming the nucleus to be stationary. By separation of variables, obtain the radial equation. Calculate the expectation value of r related to the wave function $\Psi(r)=\left(\pi a^3\right)^{-\frac{1}{2}} e^{r/a}$. 1+2+2

d) What do you mean by Normal and Anomalous Zeeman effect? Find out the different states in L-S coupling scheme for a two-electron atom. Given $l_1 = 3$ and $l_2 = 1$. 2+3

SECTION–III

3. Answer any **one** question: 10×1=10

a) i) The ground state wave function of a system in spherical polar coordinates is given by $\Psi(r, \theta, \phi)=\frac{A}{r}\left(e^{-\alpha r}-e^{-\beta r}\right)$, where A, α, β are constants. Determine A as a function of α, β so as to normalize the wave function.

ii) An electron with a kinetic energy 10 eV is moving from left to right along the x-axis. The potential energy $V = 0$ for $x < 0$ and $V = 20$ eV for $x > 0$.

A) Write down Schrödinger's equation for the regions $x < 0$ and $x > 0$.

B) Calculate the reflection coefficient.

C) Sketch the solutions of the wave equations in both the regions.

iii) Prove that a Hermitian operator has real Eigen values. 4+4+2

b) i) An electron is described by the wave function:

$$\Psi(x)=\begin{cases} 0 & for\ \ x<0 \\ Ce^{-x}\left(1-e^{-x}\right) & for\ \ x>0 \end{cases}$$

where C is a constant.

A) Determine the value of C that normalizes $\Psi(x)$.

B) Where is the electron most likely to be found?

C) Calculate the average position $\langle x \rangle$ for the electron. Compare this result with the most likely position, and comment on the difference.

ii) Calculate $\langle x \rangle$, $\langle x^2 \rangle$ for a quantum harmonic oscillator in its ground state.

(1+2+3)+(2+2)

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